CHAPTER

12 LINEAR PROGRAMMING

Syllabus

Introduction, related terminology such as constraints, objective function, optimization, graphical method of solution for problems in two variables, feasible and infeasible region (bounded or unbounded), feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

In this chapter you will study

- *Components of linear programming*
- *Graphical method to solve LPP*
- *Feasible and Infeasible Solutions of LPP*

Revision Notes

Linear programming problems: Problems which minimize or maximize a linear function z subject to certain conditions determined by a set of linear inequalities with non-negative variables are known as linear programming problems.

Objective function: A linear function $z = ax + by$, where a and b are constants which has to be maximised or minimised according to a set of given conditions, is called as linear objective function.

Decision variables: In the objective function $z = ax + by$, the variables x, y are said to be decision variables.

Constraints: The restrictions in the form of inequalities on the variables of a linear programming problems are called constraints. The condition $x \geq 0$, $y \geq 0$ are known as non-negative restrictions.

Key Terms

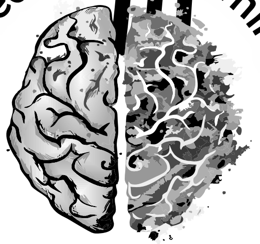
Feasible region: The common region determined by all the constraints including non-negative constraints $x, y \geq 0$ of linear programming problem is known as the feasible region.

Feasible solution: Points with in and on the boundary of the feasible region represents feasible solutions of constraints.

In the feasible region, there are infinitely many points (solutions) which satisfy the given conditions.

Theorem 1: Let R be the feasible region for a linear programming problem and let $Z = ax + by$ be the objective function. When Z has an optimal

Linear Programming



Definition

A. L.P.P. is one that is concerned with finding the optimal value (max. or min.) of a linear function of several variables (called objective function) subject to the conditions that the variables are non-negative and satisfy a set of linear inequalities (called linear constraints). Variables are sometimes called decision variables and are non-negative.

Solution of a L.P.P.

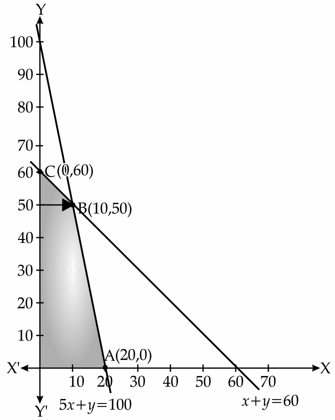
The common region determined by all the constraints including the non-negative constraint $x \geq 0, y \geq 0$ of a L.P.P. is called the feasible region (or solution region) for the problem. Points within and on the boundary of the feasible region represent feasible solutions of the constraints. Any point outside the feasible region is an infeasible solution. Any point in the feasible region that gives the optimal value (max. or min.) of the objective function is called an optimal solution.

eg : Max $Z = 250x + 75y$, subject to the

Constraints: $5x + y \leq 100$

$x + y \leq 60$

$x \geq 0, y \geq 0$ is an L.P.P.



Fundamental Theorems

Theorem 1 : Let R be the feasible region (convex polygon) for a L.P. and let $Z=ax+by$ be the objective function. When Z has an optimal value (max. or min.), where the variables x,y are subject to the constraints described by linear inequalities, this optimal value must occur at a corner point (vertex) of the feasible region,

Theorem 2: Let R be the feasible region for a L.P.P, and let $Z=ax+by$ be the objective function. If R is bounded then the objective function Z has both a max. and a min. value on R and each of these occurs at a corner point (vertex) of R.

If the feasible region is unbounded, then a max. or a min. may not exist. If it exists, it must occur at a corner point of R.

Corner point method

1. Find the feasible region of the linear programming problem and determine its corner points (vertices) either by inspection or by solving the two equations of the lines intersecting at that point.
2. Evaluate the objective function $Z = ax+by$ at each corner point. Let M and m, respectively denote the largest and smallest values of these points.
3. (i) When the feasible region is **bounded,** M and m are the maximum and minimum values of Z

   (ii) In case, the feasible region is **unbounded,** we have:
4. (a) M is the maximum value of Z, if the open half plane determined by $ax+by>M$ has no point in common with the feasible region. Otherwise, Z has no maximum value.

   (b) Similarly, m is the minimum value of Z, if the open half plane determined by $ax+by<m$ has no point in common with the feasible region. Otherwise, Z has no minimum value.

Trace the Mind Map

First Level · Second Level · Third Level value (maximum or minimum), where variables x and y are subject to constraints described by linear inequalities, the optimal value must occur at a corner point (vertex) of the feasible region.

Theorem 2: Let R be the feasible region for a linear programming problem, and let $Z = ax + by$ be the objective function. If R is bounded, then the objective function Z has both maximum and minimum values of R and each of these occurs at a corner point (vertex) of R.

However, if the feasible region is unbounded, the optimal value obtained may not be maximum or minimum.

Mnemonics

LLP parameters

N	O	C
↓	↓	↓
Non-negative variables	Objective function	Constraints

Key Facts

- Linear programming is often used for problems where no exact solution is known, for example for planning traffic flows.
- The goal of linear programming is to maximize or minimize specified objectives, such as profit or cost. This process is known as optimization.
- Linear programming is heavily used in microeconomics and company management, such as planning, product, transportation, technology and other issues, either to maximize the income or minimize the costs of a production scheme.

OBJECTIVE TYPE QUESTIONS

Multiple Choice Questions

Q. 1. A Linear Programming Problem is as follows:

Minimise $Z = 2x + y$

Subject to the constraints $x \geq 3, x \leq 9, y \geq 0$

$x - y \geq 0, x + y \leq 14$

The feasible region has

(A) 5 corner points including (0, 0) and (9, 5)

(B) 5 corner points including (7, 7) and (3, 3)

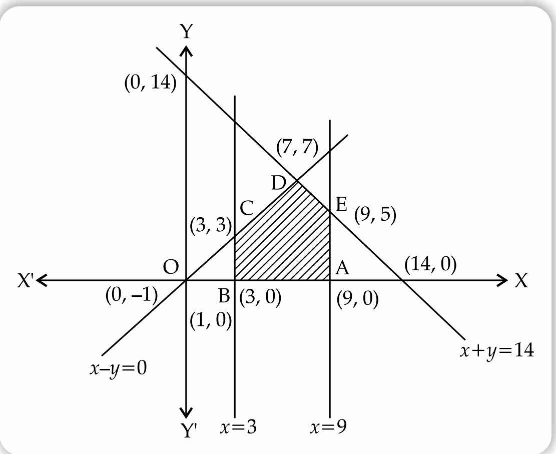
(C) 5 corner points including (14, 0) and (9, 0)

(D) 5 corner points including (3, 6) and (9, 5)

[CBSE Term-I 2021]

Ans. Option (B) is correct.

Explanation:



On plotting the constraints $x = 3$, $x = 9$, $x = y$ and $x + y = 14$, we get the following graph. From the graph given below it, clear that feasible region is ABCDEA, including corner points A(9, 0), B(3, 0), C(3, 3), D(7, 7) and E(9, 5).

Thus feasible region has 5 corner points including (7, 7) and (3, 3).

Q. 2. The corner points of the feasible region for a Linear Programming problem are P(0, 5), Q(1, 5), R(4, 2) and S(12, 0). The minimum value of the objective function Z = $2x + 5y$ is at the point

(A) P **(B)** Q

(C) R **(D)** S [CBSE Term-I 2021]

Ans. Option (C) is correct.

Explanation:

Corner Points	Value of $Z = 2x + 5y$
P(0, 5)	Z = 2(0) + 5(5) = 25
Q(1, 5)	Z = 2(1) + 5(5) = 27
R(4, 2)	Z = 2(4) + 5(2) = 18 → Minimum
S(12, 0)	Z = 2(12) + 5(0) = 24

Thus, minimum value of Z occurs ar R(4, 2).

Q. 3. A Linear Programming Problem is as follows:
Maximise/Minimise objective function Z = $2x - y + 5$

Subject to the constraints

$$3x + 4y \leq 60$$

$$x + 3y \leq 30$$

$$x \leq 0, y \geq 0$$

In the corner points of the feasible region are A(0, 10), B(12, 6), C(20, 0) and O(0, 0), then which of the following is true?

(A) Maximum value of Z is 40

(B) Minimum value of Z is –5

(C) Difference of maximum and minimum values of Z is 35

(D) At two corner points, value of Z are equal

[CBSE Term-I 2021]

Ans. Option (B) is correct.

Explanation:

Corner Points	Value of $Z = 2x - y + 5$
A(0, 10)	Z = 2(0) – 10 + 5 = –5 (Minimum)
B(12, 6)	Z = 2(12) – 6 + 5 = 23
C(20, 0)	Z = 2(20) – 0 + 5 = 45 (Minimum)
O(0, 0)	Z = 0(0) – 0 + 5 = 5

So the minimum value of Z is –5.

Q. 4. The corner points of the feasible region determined by a set of constraints (linear inequalities) are P(0, 5), Q(3, 5), R(5, 0) and S(4, 1) and the objective function is Z = $ax + 2by$ where a, $b > 0$. The condition on a and b such that the maximum Z occurs at Q and S is

(A) $a - 5b = 0$ **(B)** $a - 3b = 0$

(C) $a - 2b = 0$ **(D)** $a - 8b = 0$

[CBSE Term-I 2021]

Ans. Option (D) is correct.

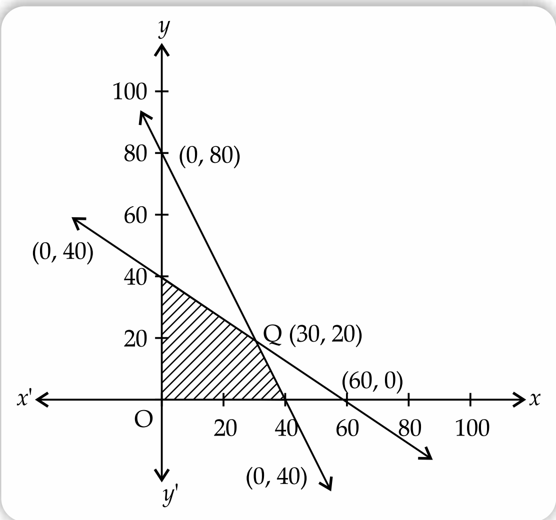
Explanation: Given, Max. Z = $ax + 2by$

Max. value of Z on Q(3, 5) = Max. value of Z on S(4, 1)

$\Rightarrow \quad 3a + 10b = 4a + 2b$

$\Rightarrow \quad a - 8b = 0$

Q. 5. For an L.P.P. the objective function is Z = $4x + 3y$, and the feasible region determined by a set of constraints (linear in equations) is shown in the graph.



Which one of the following statements is true?

(A) Maximum value of Z is at R.

(B) Maximum value of Z is at Q.

(C) Value of Z at R is less than the value at P.

(D) Value of Z at Q is less than the value at R.

[CBSE Term-I 2021]

Ans. Option (B) is correct.

Explanation:

Corner Points of Feasible Region	Value of Z = ($Z = 4x + 3y$)
O(0, 0)	Z = 4(0) + 3(0) = 0
P(0, 40)	Z = 4(0) + 3(40) = 120
Q(30, 20)	Z = 4(30) + 3(20) = 180 (Minimum)
R(40, 0)	Z = 4(40) + 3(0) = 160

Thus, Maximum value of z is at Q, which is 180.

Q. 6. The corner points of the feasible region determined by the system of linear inequalities are (0, 0), (4, 0), (2, 4), and (0, 5). If the maximum value of $Z = ax + by$, where a, $b > 0$ occurs at both (2, 4) and (4, 0), then

(A) $a = 2b$ **(B)** $2a = b$

(C) $a = b$ **(D)** $3a = b$

[CBSE Delhi Set-II 2020]

Ans. Option (A) is correct.

Explanation:

Topper Answer, 2020

Sol. $2a + 4b = 4a$

$\Rightarrow 2a = 4b$

$a = 2b$

Q. 7. The corner points of the feasible region determined by the system of linear constraints are (0, 0), (0, 40), (20, 40), (60, 20), (60, 0). The objective function is

$$Z = 4x + 3y.$$

Compare the quantity in Column A and Column B

Column A	Column B
Maximum of Z	325

(A) The quantity in column A is greater.

(B) The quantity in column B is greater.

(C) The two quantities are equal.

(D) The relationship cannot be determined on the basis of the information supplied.

Ans. Option (B) is correct.

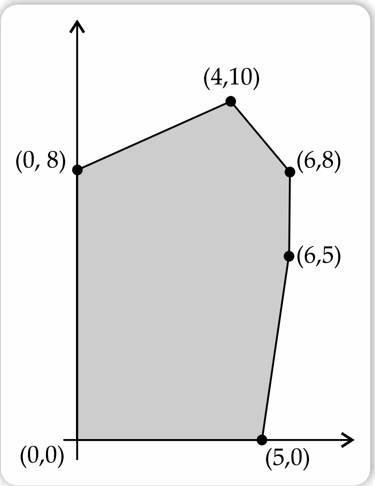
Explanation:

Corner points	Corresponding value of $Z = 4x + 3y$
(0, 0)	0
(0, 40)	120
(20, 40)	200
(60, 20)	300 ← Maximum
(60, 0)	240

Hence, maximum value of Z = 300 < 325

So, the quantity in column B is greater.

Q. 8. **The feasible solution for a LPP is shown in given figure. Let $Z = 3x - 4y$ be the objective function Minimum of Z occurs at**



(A) (0, 0) **(B)** (0, 8)

(C) (5, 0) **(D)** (4, 10)

Q. 9. **Refer to Q.8 maximum of Z occurs at**

(A) (5, 0) **(B)** (6, 5)

(C) (6, 8) **(D)** (4, 10)

Ans. **Option (A) is correct.**

Explanation: Maximum of Z occurs at (5, 0).

Q. 10. **Refer to Q.8 of multiple choice questions, (Maximum value of Z + Minimum value of Z) is equal to**

(A) 13 **(B)** 1

(C) –13 **(D)** –17

Ans. **Option (D) is correct.**

Explanation: Maximum value of Z + Minimum value of Z = 15 − 32 = −17

SUBJECTIVE TYPE QUESTIONS

Short Answer Type Questions-I (2 marks each)

Q. 1. **Two tailors, A and B, earn ₹ 300 and ₹ 400 per day respectively. A can stitch 6 shirts and 4 pairs of trousers while B can stitch 10 shirts and 4 pairs of trousers per day. To find how many days should each of them work and if it is desired to produce at least 60 shirts and 32 pairs of trousers at a minimum labour cost, formulate this as an LPP.**

R&U [O.D. Set-I, 2017]

Sol. Let A works for x day and B for y days.

$\therefore$ L.P.P. is Minimize $C = 300x + 400y$ ½

Subject to : $\begin{cases} 6x + 10y \geq 60 \\ 4x + 4y \geq 32 \\ x \geq 0, y \geq 0 \end{cases}$ 1½

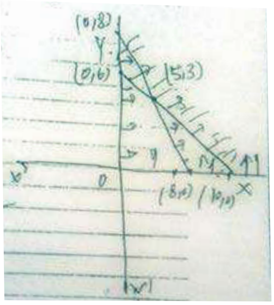
[CBSE Marking Scheme 2017]

Detailed Solution:

Topper Answer, 2017

Sol.

	Shirts	Trousers	Payment per day
(x) A	6	4	₹300
(y) B	10	4	₹ 400
	60	32	



Let A for work for x and B for y days.

To Minimize : $Z = 300x + 400y$

Constraints : $6x + 10y \geqslant 60$

$4x + 4y \geqslant 32$

$x \geqslant 0$

$y \geqslant 0$

PQR is the solution.

$P(5,3)$ $z = 300(5) + 400(3) = ₹2700$

Q (0,8)	z = 400 (8) = ₹ 3200	
R (10,0)	z = 300 (10) = ₹ 3000	

Hence z is minimum when A works for 5 days and B for 3 days.

Q. 2. A small firm manufactures necklaces and bracelets. The total number of necklaces and bracelets that it can handle per day is at most 24. It takes one hour to make a bracelet and half an hour to make a necklace. The maximum number of hours available per day is 16. If the profit on a necklace is ₹ 100 and that on a bracelet is ₹ 300. Formulate on L.P.P. for finding how many of each should be produced daily to maximize the profit ? It is being given that at least one of each must be produced. R&U [Delhi 2017]

Q. 3. A firm has to transport at least 1200 packages daily using large vans which carry 200 packages each and small vans which can take 80 packages each. The cost for engaging each large van is ₹ 400 and each small van is ₹ 200. Not more than ₹ 3,000 is to be spent daily on the job and the number of large vans cannot exceed the number of small vans. Formulate this problem as a LPP given that the objective is to minimize cost. R&U [Delhi Comptt. 2017]

Q. 4. If a 20 year old girl drives her car at 25 km/h, she has to spend ₹ 4/km on petrol. If she drives her car at 40 km/h, the petrol cost increases to ₹ 5/km. She has ₹ 200 to spend on petrol and wishes to find the maximum distance she can travel within one hour. Express the above problem as a Linear Programming Problem. R&U [SQP Dec. 2016-17]

Sol. Let the distance covered with speed of
25 km/h = x km
and the distance covered with speed of
40 km/h = y km ½
Total distance covered = z km
The L.P.P. of the above problem, therefore, is
Maximize $z = x + y$ ½
Subject to constraints
$4x + 5y \le 200$
$\frac{x}{50} + \frac{y}{40} \le 1$ ½
$x \ge 0, y \ge 0$ ½
[CBSE SQP Marking Scheme 2016]

Commonly Made Error

- Some students take incorrect inequality sign hence they get incorrect feasible region.

Answering Tip

- Students should sketch the correct equation with proper sign $ax + by \le c$ or $ax + by \ge c$ according to the problem.

Short Answer Type Questions-II (3 or 4 marks each)

Q. 1. A furniture trader deals in only two items — chairs and tables. He has ₹50,000 to invest and a space to store at most 35 items. A chair costs him ₹1,000 and a table costs him ₹2,000. The trader earns a profit of ₹150 and ₹250 on a chair and table, respectively. Formulate the above problem as a LPP to maximise the profit and solve it graphically. A I [CBSE Delhi Set-II 2020]

Topper Answer, 2020

Sol. let the trader buy x number of chairs
and y number of tables

He has to maximise profit Z = 150x + 250y
He has only ₹ 50000 to invest.
∴ 1000x + 2000y ≤ 50000
⇒ x + 2y ≤ 50

Also he has a storage space of ~~ott~~ at most 35 items

$\therefore x+y \leq 35$.

$\therefore$ Required LPP is

Maximize $Z = 150x + 250y$

subject to the constraints

$x + 2y \leq 50$

$x + y \leq 35$

$x \geq 0 \quad y \geq 0$. (Minimum constraints)

converting $x+2y \leq 50$ into an equation

$x + 2y = 50$

x	0	50
y	25	0

(0,0) satisfy the inequality.

converting $x+y \leq 35$ into an equation

$x + y = 35$

x	0	35
y	35	0

(0,0) satisfies the inequality

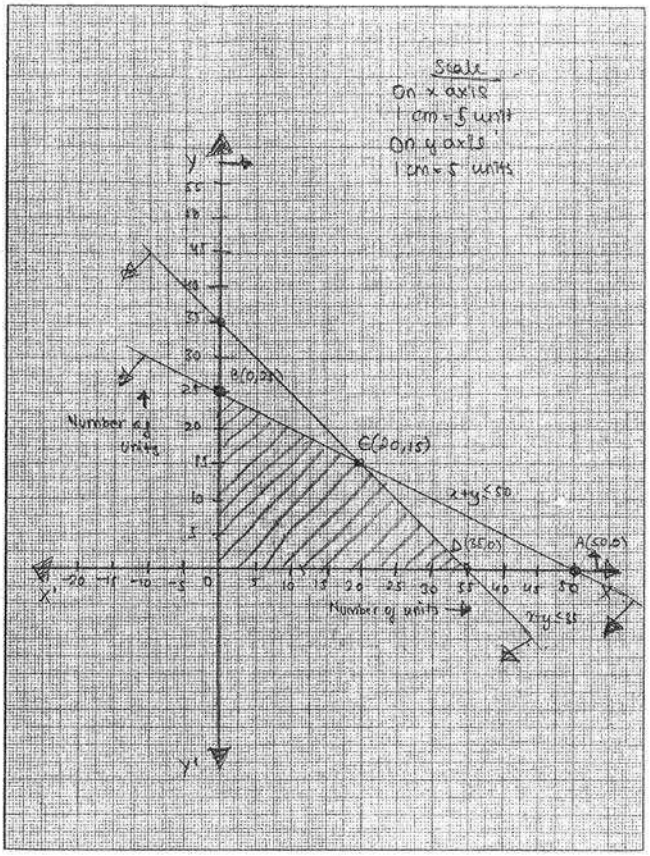
solving both equations simultaneously we get E(20,15)

Feasible area is represented by BODE .

Corner Points	$Z = 150x + 250y$	
O(0,0)	0+0=0	= 0
D(35,0)	35×150	= 5250
B(0,25)	25×250	= 6250
E(20,15)	20×150+15×250	= 6750 ⇒ Maximum

for maximum profit he will trade in 20 items of chairs

and 15 items of tables.



Q. 2. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type *A* requires 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type *B* require 8 minutes each for cutting and 8 minutes each for assembling. Given that total time for cutting is 3 hours 20 minutes and for assembling 4 hours. The profit for type *A* souvenir is ₹ 100 each and for type *B* souvenir, profit is ₹ 120 each. How many souvenirs of each type should the company manufacture in order to maximize the profit ? Formulate the problem as an LPP and solve it graphically.

R&U **[CBSE Delhi Set I, II, III-2020]**

Sol. Let the company manufacture 'x' number of souvenirs of Type A

And, 'y' number of souvenirs of Type B

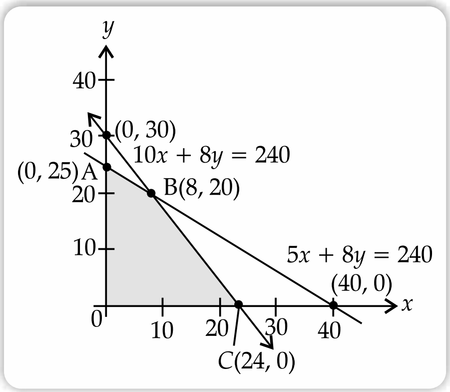
∴ LPP is: Maximise $P = 100x + 120y$ ½

subject to $5x + 8y \leq 200$

$10x + 8y \leq 240$

$x \geq 0, y \geq 0$ **1**

Correct Graph **1**



$P(A) = ₹3,000$

$P(B) = ₹3,200$ (Max.)

$P(C) = ₹2,400$

∴ For maximum profit,

No. of souvenirs of Type A = 8

No. of souvenirs of Type B = 20 ½

[CBSE Marking Scheme 2020] (Modified)

Detailed Solution:

Suppose number of souvenirs of type *A* and type *B* are x and y respectively.

Then, the LPP is as follows:

Maximize $Z = 100x + 120y$

Subject to $5x + 8y \leq 200$

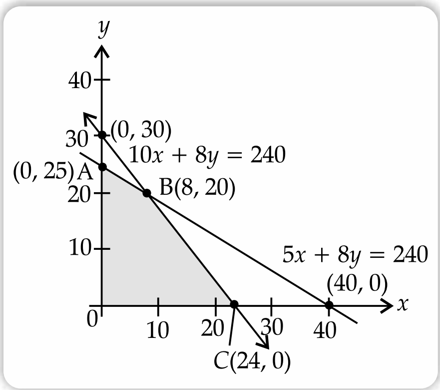
$$10x + 8y \le 240$$

and $\quad x, y \ge 0$

To solve the LPP graphically first we convert inequalities into equations and draw the corresponding lines.

Then,

Corner points	Value of Z in ₹
A (0, 25)	3000
B (8, 20)	3200 maximum
C (24, 0)	2400



Clearly, maximum profit is obtained when 8 souvenirs of type A and 20 souvenirs of type B is manufactured.

Then Maximum profit $= 100 \times 8 + 120 \times 20$

$= ₹3200$

Commonly Made Error

- Mostly students carelessly ignore the hour to minute conversion and go wrong.

Answering Tip

- Read the question carefully and get used with the unit conversions.

Q. 3. A manufacturer has three machines I, II and III installed in his factory. Machine I and II are capable of being operated for atmost 12 hours whereas machine III must be operated for atleast 5 hours a day. He produces only two items M and N each requiring the use of all the three machines.

The number of hours required for producing 1 unit of M and N on three machines are given in the following table:

Items	Number of hours required on machines		
	I	II	III
M	1	2	1
N	2	1	1.25

He makes a profit of ₹ 600 and ₹ 400 on one unit of items M and N respectively. How many units of each item should he produce so as to maximize his profit assuming that he can sell all the items that he produced. What will be the maximum profit ?

R&U [CBSE OD Set I, II, III-2020]

Q. 4. Two tailors A and B earn ₹ 150 and ₹ 200 per day respectively. A can stitch 6 shirts and 4 pants per day, while B can stitch 10 shirts and 4 pants per day. Form a L.P.P. to minimize the labour cost of produce (stitch) at least 60 shirts and 32 pants and solve it graphically. AI A [CBSE SQP-2020]

Sol. Let tailor A works for x days and tailor B works for y days ½

Objective function:

To minimize labour cost Z = $150x + 200y$ (in ₹)

Subject to constraints

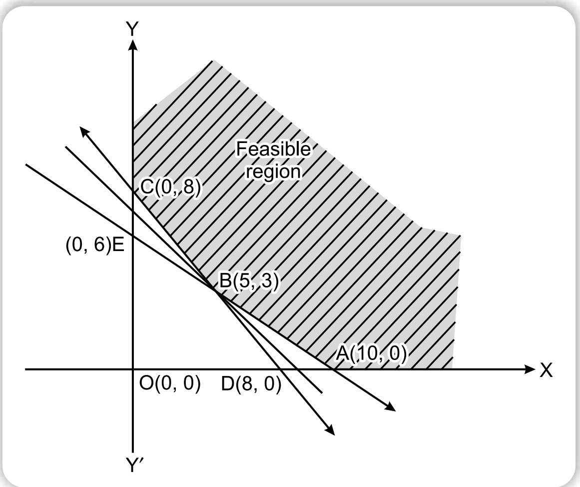
$$6x + 10y \ge 60 \text{ i.e. } 3x + 5y \ge 30$$
$$4x + 4y \ge 32 \text{ i.e. } x + y \ge 8$$
$$x \ge 0, y \ge 0 \qquad 1$$

Consider equations to draw the graph and then we will shade feasible region

$$3x + 5y = 30$$
$$x + y = 8$$

Corner points of feasible region are $A(10, 0)$, $B(5, 3)$ and C(0, 8)

Value of Z at these corner points



1

So minimum value of Z is ₹ 1350 when tailor A works for 5 days and tailor B works for 3 days.

To check draw $150x + 200y < 1350$ i.e. $3x + 4y < 27$

As there is no region common with feasible region so minimum value is ₹ 1350. ½

[CBSE SQP Marking Scheme, 2020] (Modified)

Commonly Made Error

- Mostly students fail to draw the optimal line correctly.

Answering Tip

- Practice more problems on LPP involving unbounded feasible region.

Q. 5. **Maximise $Z = x + 2y$**

Subject to the constraints

$$x + 2y \geq 100$$
$$2x - y \leq 0$$
$$2x + y \leq 200$$
$$x, y \geq 0$$

Solve the above LPP graphically.

AP [O.D. Set-I, 2017]

Q. 6. **Solve the following L.P.P. graphically:**

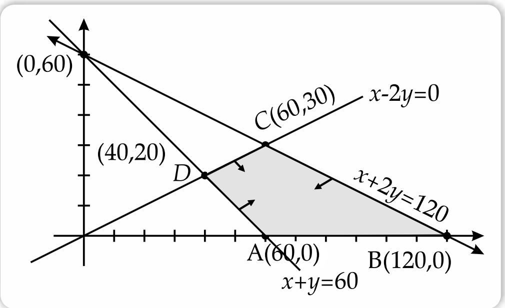
Minimise $Z = 5x + 10y$

Subject to Constraints $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$

and $x, y \geq 0$

AP [Delhi I, II, III 2017]

Sol.



Correct graph of 3 lines **1**

Correct shade of 3 lines **1**

$$Z = 5x + 10y$$
$$Z|_{A\,(60,\,0)} = 300$$
$$Z|_{B\,(120,\,0)} = 600$$
$$Z|_{C\,(60,\,30)} = 600$$
$$Z|_{D\,(40,\,20)} = 400$$

Minimum value of $Z = 300$ at $x = 60, y = 0$ **1**

[CBSE Marking Scheme, 2017] (Modified)

Q. 7. **Solve the following L.P.P. graphically**

Maximise $Z = 4x + y$

Subject to following constraints $x + y \leq 50,$ $3x + y \leq 90,$ $x \geq 10$, $x, y \geq 0$

AP [Delhi I, II, III 2017]

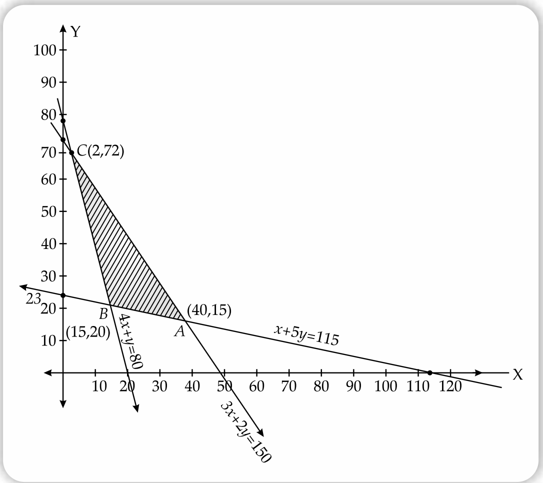
Q. 8. **Solve the following Linear Programming problem graphically:**

Minimize: $Z = 6x + 3y$

Subject to the constraints: $\begin{cases} 4x + y \geq 80 \\ x + 5y \geq 115 \\ 3x + 2y, \leq 150 \\ x \geq 0, y \geq 0 \end{cases}$

AP [Delhi Comptt. 2017]

Sol.



Correct lines		1
Correct shading		½
Corner points	Value of Z	
A (40, 15)	285	
B (15, 20)	150 → minimum	1
C (2, 72)	228	
minimum	$Z = 150$	
when	$x = 15, y = 20$	½

[CBSE Marking Scheme, 2017] (Modified)

Long Answer Type Questions (5 or 6 marks each)

Q. 1. **Solve the following linear programming problem (L.P.P) graphically.**

Maximize Z = $x + 2y$

subject to constraints;

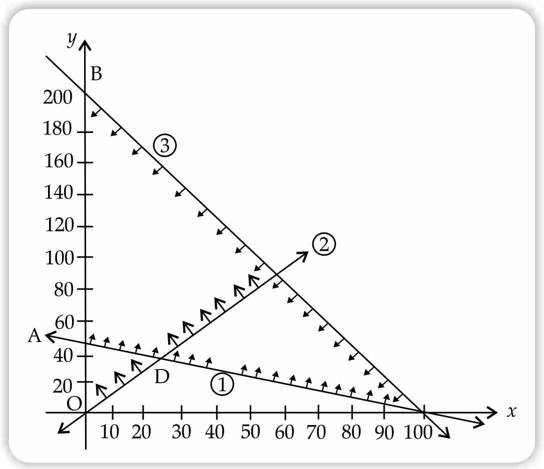
$$x + 2y \geq 100$$
$$2x - y \leq 0$$
$$2x + y \leq 200$$
$$x, y \geq 0$$

AI R&U [CBSE SQP 2020-21]

Sol. Max $Z = 3x + y$

Subject to

$$x + 2y \geq 100 \quad ...(i)$$
$$2x - y \leq 0 \quad ...(ii)$$
$$2x + y \leq 200 \quad ...(iii)$$
$$x \geq 0$$
$$y \geq 0,$$



3

Corner Points	$Z = 3x + y$
A(0, 50)	50
B(0, 200)	200
C(50, 100)	250
D(20, 40)	100

1

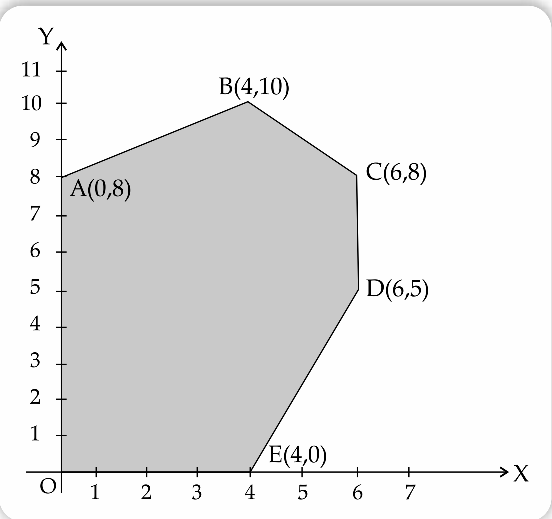
Max $z = 250$

at $x = 50$

$y = 100$ 1

[CBSE SQP Marking Scheme 2020-21]

Q. 2. The corner points of the feasible region determined by the system of linear constraints are as shown below:



Answer each of the following:

(i) Let Z = $3x - 4y$ be the objective function. Find the maximum and minimum value of Z and also the corresponding points at which the maximum and minimum value occurs.

(ii) Let Z = $px + qy$, where $p, q > 0$ be the objective function. Find the condition on p and q so that the maximum value of Z occurs at B(4, 10) and C(6,8). Also mention the number of optimal solutions in this case. U **[CBSE SQP 2020-21]**

Q. 3. A manufacturer has employed 5 skilled men and 10 semi-skilled men and makes two models *A* and *B* of an article. The making of one item of model *A* requires 2 hours work by a skilled man and 2 hours work by a semi-skilled man. One item of model *B* requires 1 hour by a skilled man 3 hours by a semi-skilled man. No man is expected to work more than 8 hours per day. The manufacturer's profit on an item of model *A* is ₹15 and on an item of model *B* is ₹10. How many of items of each model should be made per day in order to maximize daily profit ? Formulate the above LPP and solve it graphically and find the maximum profit. A **[CBSE Delhi Set III-2019]**

Sol. Let number of items produced of model *A* be x and that of model *B* be y.

LPP is:

Maximize, profit $z = 15x + 10y$ 1

subject to

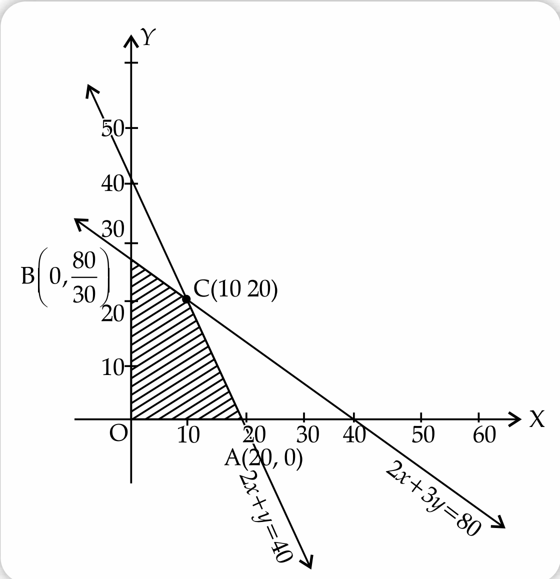
$$\left.\begin{array}{ll} 2x + y \le 5(8) & i.e.,\ \ 2x + y \le 40 \\ 2x + 3y \le 10(8) & i.e.,\ \ 2x + 3y \le 80 \\ x \ge 0, y \ge 0 & \end{array}\right\}$$ 1

Corner point	$z = 15x + 10y$	
$A(20, 0)$	300	
$B\left(0, \dfrac{80}{3}\right)$	$\dfrac{800}{3} = 266.6$	½
$C(10, 20)$	350 ← maximum	

Maximum profit = ₹ 350

when $x = 10, y = 20$. ½

Correct Figure 2



If a student has interpreted the language of the question in a different way, then the LPP will be of the type:

Maximise profit $z = 15x + 10y$

Subject to $2x + y \le 8$

$2x + 3y \le 8$

$x \ge 0, y \ge 0$

This is be accepted and marks may be given accordingly.

[CBSE Marking Scheme, 2019] (Modified)

Detailed Solution:

Topper Answer, 2019

Sol.

let x be no. of A models

y be no. of B models.

Objective $Z = 15x + 10y$ (Maximize)

Subject to constraints: $x, y \geq 0$

$2x + y \leq 40$ (skilled man working hrs)

$2x + 3y \leq 80$ (Semi-skilled man working hrs)

$2x + y \leq 40$

$2x + y = 40$

x	0	20
y	40	0

Zero test : TRUE

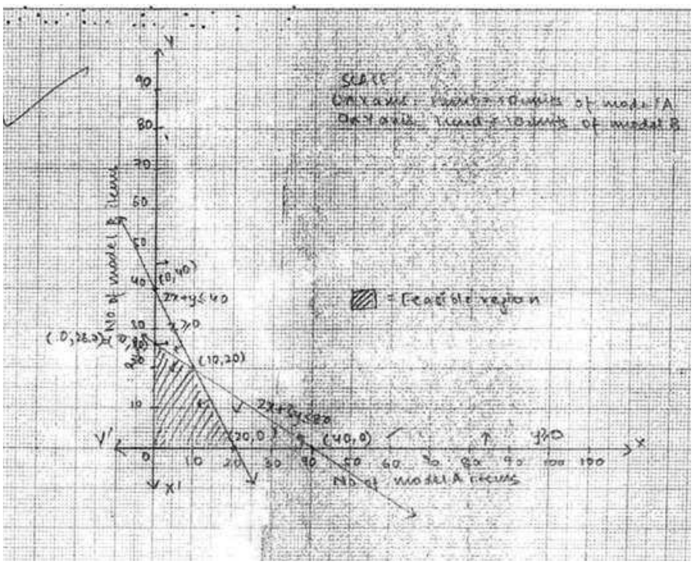
$2x + 3y \leq 80$

$2x + 3y = 80$

x	10	40
y	20	0

Zero test : TRUE

GRAPH: On graph paper.



Q. 4. A company produces two types of goods, A and B, that require gold and silver. Each unit of type A requires 3 g of silver and 1 g of gold while that of type B requires 1 g of silver and 2 g of gold. The company can use at the most 9 g of silver and 8 g of gold. If each unit of type A brings a profit of ₹40 and that of type B ₹50, find the number of units of each type that the company should produce to maximize profit. Formulate the above LPP and solve it graphically and also find the maximum profit.

A I AE [CBSE OD Set-I, 2019] [Foreign, 2017]

Q. 5. A manufacturer makes two types of toys *A* and *B*. Three machine are needed for this purpose and the time (in minutes) required for each toy on the machines is given below:

Types of Toys	Machines		
	I	II	III
A	20	10	10
B	10	20	30

The machines I, II and III are available for a maximum of 3 hours, 2 hours and 2 hours 30 minutes respectively. The profit on each toy of type A is ₹50 and that of type B is ₹60.
Formulate the above problem as a L.P.P. and solve it graphically to maximize profit.

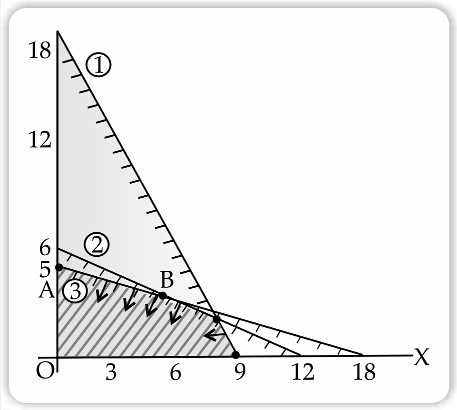
A I R&U [SQP 2018-19]

Sol. Let the manufacturer make x and y quantity of toy A and toy B respectively.

$$\text{Max } Z = 50x + 60y \quad \textbf{1}$$

Subject to

$$20x + 10y \le 180 \quad ...(1) \ \textbf{1}$$
$$10x + 20y \le 120 \quad ...(2)$$
$$10x + 30y \le 150 \quad ...(3)$$
$$x, y \le 0$$



2

Hence, Max Profit is ₹ 520, at $x = 8$ and $y = 2$ **1**

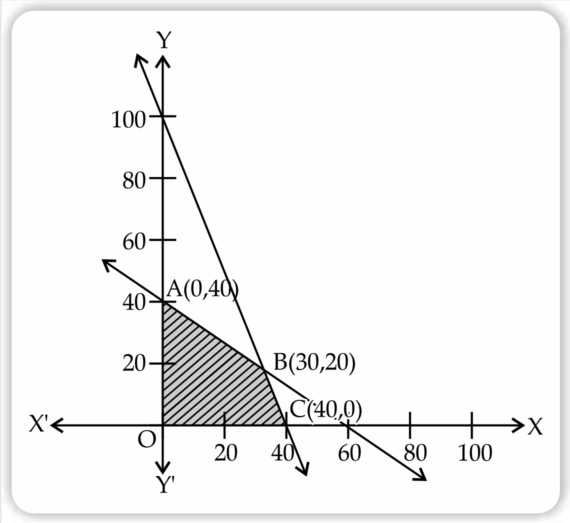
[CBSE Marking Scheme 2018] (Modified)

Q. 6. A factory manufactures two types of screws A and B, each type requiring the use of two machines, an automatic and a hand-operated. It takes 4 minutes on the automatic and 6 minutes on the hand-operated machines to manufacture a packet of screws 'A' while it takes 6 minutes on the automatic and 3 minutes on the hand-operated machine to manufacture a packet of screws 'B'. Each machine is available for at most 4 hours on any day. The manufacturer can sell a packet of screws 'A' at a profit of 70 paise and screws 'B' at a profit of ₹ 1. Assuming that he can sell all the screws he manufactures, how many packets of each type should the factory owner produce in a day in order to maximize his profit ? Formulate the above L.P.P. and solve it graphically and find the maximum profit. **E [CBSE Delhi/OD-2018]**

Sol. Let number of packets of type $A = x$

and number of packets of type $B = y$

$\therefore$ L.P.P. is: Maximize, $Z = 0.7x + y$ **1**



Subject to constraints:

$4x + 6y \le 240$ or $2x + 3y \le 120$

$6x + 3y \le 240$ or $2x + y \le 80$ **2**

$x \ge 0, y \ge 0$

Correct graph

$Z(0, 0) = 0, Z(0, 40) = 40$

$Z(40, 0) = 28, Z(30, 20) = 41$ (Max.)

$\therefore$ Max. profit ₹ 41 at $x = 30, y = 20.$ **2**

[CBSE Marking Scheme 2018] (Modified)

Detailed Solution:

Topper Answer, 2018

Sol. Let x and y be the decision variables where,
x represents the packets of Screw A and
y represents the packets of Screw B.
$Z = 70x + 100y$ {to be maximised}
Subject to constraints,

$x \geq 0$, $y \geq 0$ (non-negative constraints)

Converting above inequalities into equalities,

$4x + 6y = 240$ $\Rightarrow$ $2x + 3y = 120$

x	0	60
y	40	0

$6x + 3y = 240$ $\Rightarrow$ $2x + y = 80$

x	0	40
y	80	0

$x = 0$ and $y = 0$ (non negative constraints)

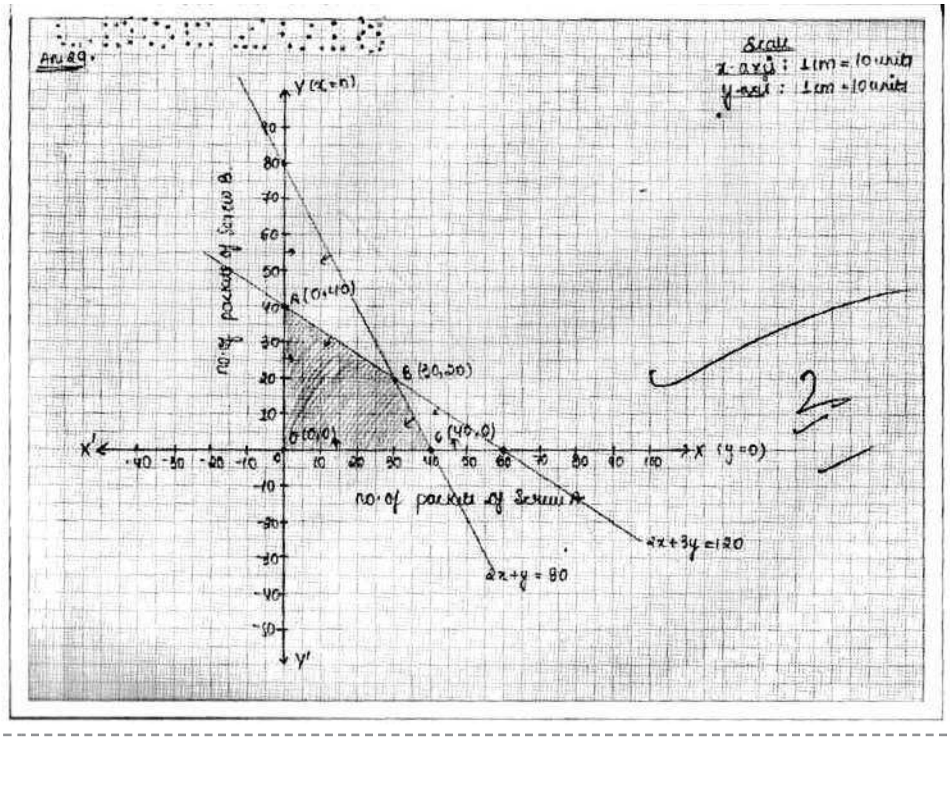
Consider a test point (0,0)

$2x + 3y \leq 120$
$0 + 0 \leq 120$
$0 < 120$
which is true

$2x + y \leq 80$
$0 + 0 \leq 80$
$0 < 80$
which is true

Corner points	$Z = 70x + 100y$
O (0,0)	$Z = 0 + 0 = 0$
A (0,40)	$Z = 70(0) + 100(40) = 4000$
B (30,20)	$Z = 70(30) + 100(20) = 4100$ $\longrightarrow$ maximum value
C (40,0)	$Z = 70(40) + 100(0) = 2800$

The factory owner must produce 30 packets of Screw A and 20 packets of Screw B to maximise his profit.
His maximum profit = 4100 paise or Rs. 41



Q. 7. One kind of cake requires 200 g of flour and 25 g of fat, and another kind of cake requires 100 g of flour and 50 g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes. Formulate the above as an LPP and solve it graphically. A I R&U [NCERT]
[Delhi Set I, II, III Comptt. 2016]
[Delhi Comptt. 2015]

Sol. Let x cakes = first kind,
y cakes = second kind, then

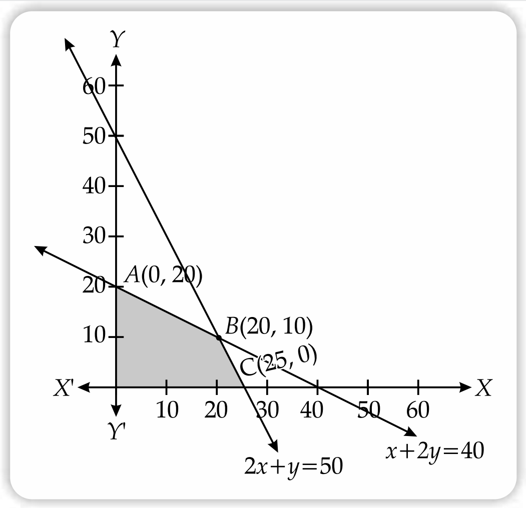
LPP is: Maximise: $Z = x + y$

Subject to:

$$200x + 100y \leq 5{,}000 \text{ or } 2x + y \leq 50$$

$$25x + 50y \leq 1{,}000 \text{ or } x + 2y \leq 40 \qquad \mathbf{2}$$

$$x, y \geq 0$$



1

Corner points	$Z = x + y$
$A(0, 20)$	$z = 20$
$B(20, 10)$	$z = 30$ (Maximum)
$C(25, 0)$	$z = 25$

$\therefore$ Maximum cakes = 30 **1**

when, Cakes of first kind = 20.

and Cakes of second kind = 10 **1**

[CBSE Marking Scheme 2016] (Modified)

Q. 8. A retired person wants to invest an amount of ₹ 50,000. His broker recommends investing in two type of bonds 'A' and 'B' yielding 10% and 9% return respectively on the invested amount. He decides to invest at least ₹ 20,000 in bond 'A' and at least ₹ 10,000 in bond 'B'. He also wants to invest at least as much in bond 'A' as in bond 'B'. Solve this linear programming problem graphically to maximise his returns. R&U [Outside Delhi 2016]

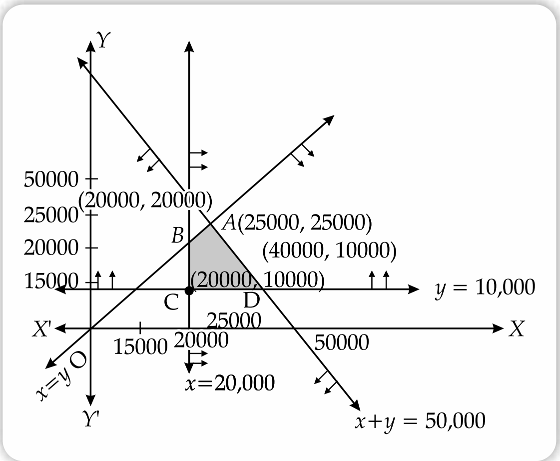
Sol. Let the investement in bond A be ₹ x and in bond B ₹ y.

Objective function is

$$Z = \frac{x}{10} + \frac{9}{100}y \qquad \mathbf{1}$$

Subject to constraints

$$x + y \leq 50{,}000;\ x \geq 20{,}000;$$
$$y \geq 10{,}000,\ x \geq y \qquad \mathbf{1}$$



2

Vertices of feasible region are A, B, C and D. **1**

Corner Points	$Z = \frac{x}{10} + \frac{9}{100}y$	Value
$A(25{,}000, 25{,}000)$	2,500 + 2,250	4,750
$B(20{,}000, 20{,}000)$	2,000 + 18,00	3,800
$C(20{,}000, 10{,}000)$	2,000 + 900	2,900
$D(40{,}000, 10{,}000)$	4,000 + 900	4,900

Return is maximum when ₹ 40,000 are invested in Bond A and ₹ 10,000 in Bond B maximum return is ₹ 4,900. **1**

Q. 9. A manufacturer produces nuts and bolts. It takes 2 hours work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 2 hours on machine B to produce a package of bolts. He earns a profit of ₹ 24 per package on nuts and ₹ 18 per package on bolts. How many packages of each should be produced each day so as to maximize his profit, if he operates his machines both for at the most 10 hours a day. Make an LPP from above and solve it graphically ?
A I R&U [OD/Delhi Comptt. 2015]

COMPETENCY BASED QUESTIONS

Case based MCQs (4 marks each)

Attempt any four sub-parts from each question. Each sub-part carries 1 mark.

I. Read the following text and answer the following questions on the basis of the same:

An aeroplane can carry a maximum of 200 passengers. A profit of ₹ 1000 is made on each executive class ticket and a profit of ₹ 600 is made on each economy class ticket. The airline reserves at least 20 seats for the executive class. However, at least 4 times as many passengers prefer to travel by economy class, than by executive class. It is given that the number of executive class tickets is x and that of economy class tickets is y.

Q. 1. The maximum value of $x + y$ is_______.

(A) 100 **(B)** 200

(C) 20 **(D)** 80

Ans. Option (B) is correct.

Q. 2. The relation between x and y is _______.

(A) $x < y$ **(B)** $y > 80$

(C) $x \geq 4y$ **(D)** $y \geq 4x$

Ans. Option (D) is correct.

Q. 3. Which among these is not a constraint for this LPP?

(A) $x \geq 0$ **(B)** $x + y \geq 200$

(C) $x \geq 80$ **(D)** $4x - y \leq 0$

Ans. Option (C) is correct.

Q. 4. The profit when $x = 20$ and $y = 80$ is _______.

(A) ₹60,000 **(B)** ₹68,000

(C) ₹64,000 **(D)** ₹1,36,000

Ans. Option (B) is correct.

Q. 5. The maximum profit is ₹ _______.

(A) 1,36,000 **(B)** 1,28,000

(C) 68,000 **(D)** 1,20,000

Ans. Option (A) is correct.

Explanation:

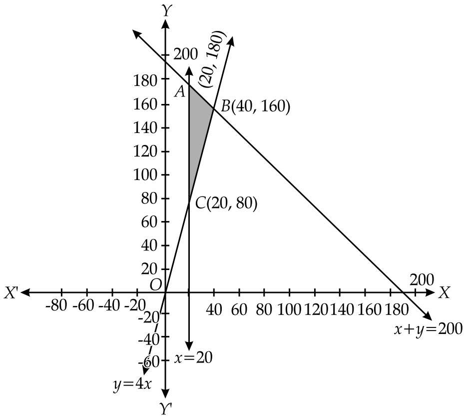
Objective function:

$$\text{Maximise } Z = 1000x + 600y$$

Constraints: $x + y \geq 200$

$y \geq 20, x \geq 0$

$y \geq 4x$



The corner points are $A(20, 180)$, $B(40, 160)$, $C(20, 80)$

Evaluating the objective function

$Z = 1,000x + 600y$ at A, B and C

At $A(20, 180), Z = 1,000 \times 20 + 600 \times 180$
$= 20,000 + 1,08,000$
$=$ ₹ 1,28,000

At $B(40, 160), Z = 1,000 \times 40 + 600 \times 160$
$= 40,000 + 96,000$
$=$ ₹ 1,36,000 (max.)

At $C(20, 80), Z = 1000 \times 20 + 600 \times 80$
$= 20,000 + 48,000$
$=$ ₹ 68,000

or Z is maximum, when $x = 40$, $y = 160$.

or 40 tickets of executive class and 160 tickets of economy class should be sold to get the maximum profit of ₹ 1,36,000.

II. Read the following text and answer the following questions on the basis of the same:

A dealer in rural area wishes to purchase a number of sewing machines. He has only ₹5,760 to invest and has space for at most 20 items for storage. An electronic sewing machine cost him ₹360 and a manually operated sewing machine ₹240. He can sell an electronic sewing machine at a profit of ₹22 and a manually operated machine at a profit of ₹18. Assume that the electronic sewing machines he can sell is x and that of manually operated machines is y.

Q. 1. The objective function is _______.

(A) Maximise $Z = 360x + 240y$

(B) Maximise $Z = 22x + 18y$

(C) Minimise $Z = 360x + 240y$

(D) Minimise $Z = 22x + 18y$

Ans. Option (B) is correct.

Q. 2. The maximum value of $x + y$ is _______.

(A) 5760 **(B)** 18

(C) 22 **(D)** 20

Ans. Option (D) is correct.

Q. 3. Which of the following is not a constraint?

(A) $x + y \geq 20$

(B) $360x + 240y \leq 5{,}760$

(C) $x \geq 0$

(D) $y \geq 0$

Ans. Option (A) is correct.

Q. 4. The profit is maximum when (x, y) = _______.

(A) (5, 15) **(B)** (8, 12)

(C) (12, 8) **(D)** (15, 5)

Ans. Option (B) is correct.

Q. 5. The maximum profit is ₹ _______.

(A) 5,760 **(B)** 392

(C) 362 **(D)** 290

Ans. Option (B) is correct.

Explanation:

Objective function :

Maximise $Z = 22x + 18y$

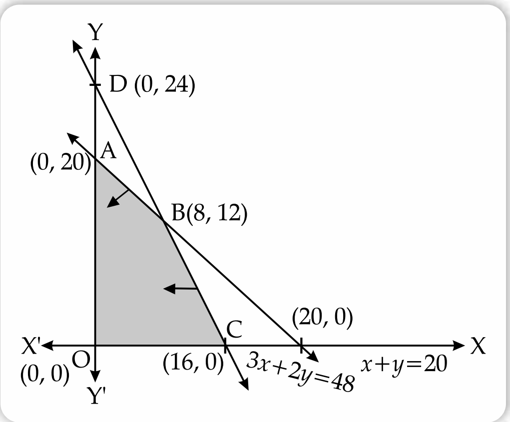
Constraints :

$$x + y \leq 20$$

$$360x + 240y \leq 5{,}760$$

Or $3x + 2y \leq 48$

$$x \geq 0, \quad y \geq 0$$



Vertices of feasible region are :

$A(0, 20)$, $B(8, 12)$, $C(16, 0)$ & $O(0, 0)$

$P(A) = 360$, $P(B) = 392$, $P(C) = 352$

$\therefore$ For Maximum P, Electronic machines = 8, and Manual machines = 12. Max. profit ₹392

Case based Subjective Questions (4 marks each)

I. Read the following text and answer the following questions on the basis of the same:

(Each Sub Part Carries 2 Marks)

Seema rides her bike at 25 km/h. She has to spend ₹2 per km on diesel and if she rides it at faster speed of 40 km/h, the diesel cost increases to ₹5 per km. She has ₹100 to spend on diesel. Let she travels x km with speed 25 km/h and y km with speed 40 km/h. It is given that objective function is Max. $Z = x + y$.

Q. 1. Construct the LPP and draw the graph for feasible region.

Sol. Required LPP is :

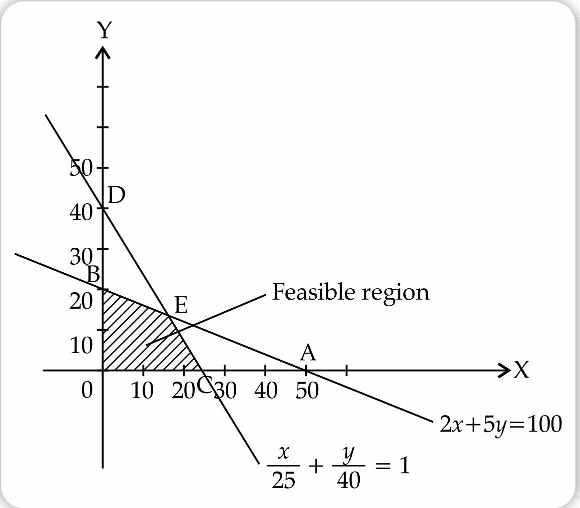
$$\text{Max } Z = x + y$$

Subject to constraints :

$$2x + 5y \leq 100$$

$$\frac{x}{25} + \frac{y}{40} \leq 1$$

$$x, y \geq 0$$ **1**



1

Q. 2. Write the corner points of feasible region and find maximum value of objective function.

Sol. Intersection point of $2x + 5y = 100$ and $\frac{x}{25}+\frac{y}{40} = 1$ is E.

On solving the equations, we get the coordinates of E are $\left(\frac{50}{3},\frac{40}{3}\right)$.

So, corner points of LPP are :

O(0, 0), B(0, 20), C(25, 0), E$\left(\frac{50}{3},\frac{40}{3}\right)$ **1**

Corner points	Value of $z = x + y$
(0, 0)	0
(0, 20)	20
(25, 0)	25
$\left(\frac{50}{3},\frac{40}{3}\right)$	30 → Maximum

Thus, maximum value of Z is 30. **1**

II. Read the following text and answer the following questions on the basis of the same:

Let R be the feasible region (convex polygon) for a linear programming problem and let $Z = ax + by$ be the objective function. When Z has an optimal value (maximum or minimum), where the variables x and y are subject to constraints described by linear inequalities, this optimal value must occur at a corner points (vertex) of the feasible region.

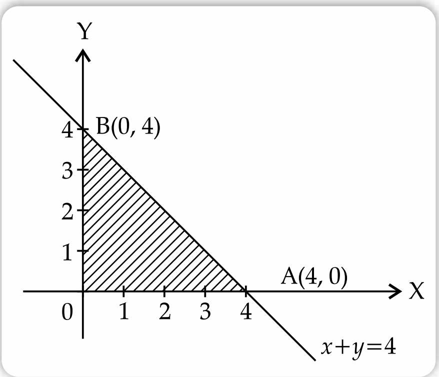
Q. 1. Solve the following LPP graphically :

Max. $Z = 2x + 3y$

subject to $x + y \leq 4$

$x \geq 0, y \geq 0$

Sol. Graph of the given LPP is shown as follows.



1

Corner points of feasible region are :
O(0, 0), A(4, 0) and B(0, 4)

Corner points	Value of Z
0(0, 0)	0
A(4, 0)	8
B(0, 4)	12 → Maximum

∴ Maximum value of Z is 12. **1**

Q. 2. Draw the graph of given LPP and find the corner points of feasible region.

Minimize $z = 200x + 500y$

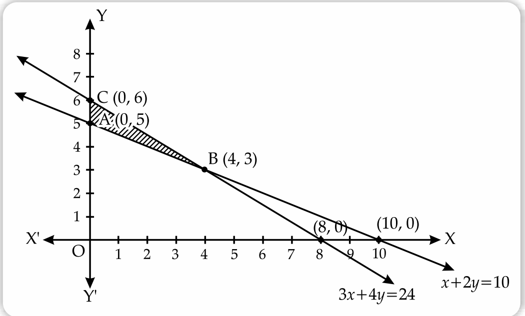
Subject to constraints :

$x + 2y \geq 10$

$3x + 4y \leq 24$

$x \geq 0, y \geq 0$

Sol.



1

The corner points of feasible region are :
A(0, 5), B(0, 6) and B(4, 3). **1**
[Coordinates of point B is obtained by solving equations $x + 2y = 10$ and $3x + 4y = 24$ simultaneously]

Solutions for Practice Questions

Multiple Choice Questions

8. Option (B) is correct.

Explanation:

Corner points	Corresponding value of $Z = 3x - 4y$
(0, 0)	0
(5, 0)	15 ← Maximum
(6, 5)	–2
(6, 8)	–14
(4, 10)	–28
(0, 8)	–32 ← Minimum

Hence, the minimum of Z occurs at (0, 8) and its minimum value is (–32).

Short Answer Type Questions-I

2. Let x necklaces and y bracelets are manufactured

$\therefore$ L.P.P. is Maximize profit,

$$P = 100x + 300y \qquad ½$$

Subject to constraints

$x + y \leq 24$

$\frac{1}{2}x + y \leq 16$ or $x + 2y \leq 32$ ½ + ½ + ½

$x, y, \geq 1$

[CBSE Marking Scheme, 2017]

3. Let number of large vans = x

and number of small vans = y

Minimize cost $z = 400\,x + 200\,y$ ½

Subject to constraints

$x + y \leq 24$

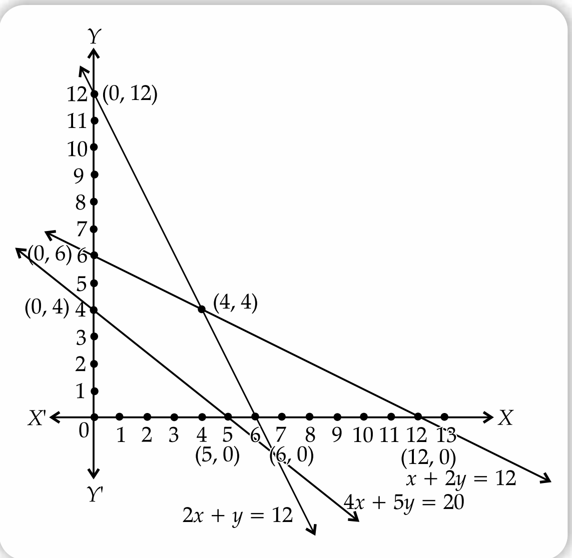
$\frac{1}{2}x + y \leq 16$ or $x = 2y \leq 32$ ½+½+½

$x, y \geq 1$

Short Answer Type Questions-II

3. Let x units of item M and y units of item N are produced.

For correct graph: **1**



Maximize $Z = 600x + 400y$

subject to

$$x + 2y \leq 12$$
$$2x + y \leq 12$$
$$x + 1.25\,y \geq 5$$
$$x \geq 0,\ y \geq 0 \qquad \mathbf{1}$$

Corner point values :

$$Z_{A(5,0)} = 3000, Z_{B(6,0)} = 3600$$
$$Z_{C(4,4)} = 4000, Z_{D(0,6)} = 2400$$
$$Z_{E(0,4)} = 1600 \qquad ½$$

$\therefore$ 4 units each of M and N must be produced to get maximum profit of ₹ 4,000 ½

[CBSE Marking Scheme 2020] (Modified)

Detailed Solution:

Let manufacturer produces x units of Product M and y units of product N.

	Product M (x)	Product N (y)	Time
Machine I	1	2	12
Machine II	2	1	12
Machine III	1	1.25	5

Subject to constraints

$$x + 2y \leq 12$$
$$2x + y \leq 12$$
$$x + 1.25y \geq 5$$

or, $4x + 5y \geq 20, x \geq 0, y \geq 0$

Maximize $Z = 600x + 400y$

L_1: $x + 2y = 12$

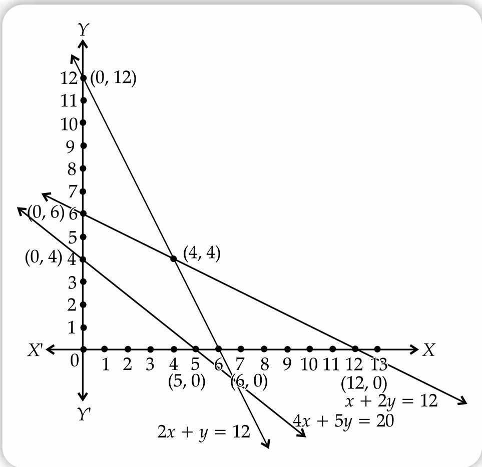
x	0	12	4
y	6	0	4

L_2: $2x + y = 12$

x	0	6	4
y	12	0	4

L_3: $4x + 5y = 20$

x	0	5
y	4	0



Vertices of feasible region are (0, 4), (5, 0), (6, 0), (4, 4) and (0, 6)

(x, y)	$Z = 600x + 400y$
(0, 4)	Z = 1600
(5, 0)	Z = 3000
(6, 0)	Z = 3600
(4, 4)	Z = 4000 (Max.)
(0, 6)	Z = 2400

Maximum Value of Z = 4000 at $x = 4$ and $y = 4$

Commonly Made Error

- Since there are three lines the students fail to identify the correct feasible region.

Answering Tip

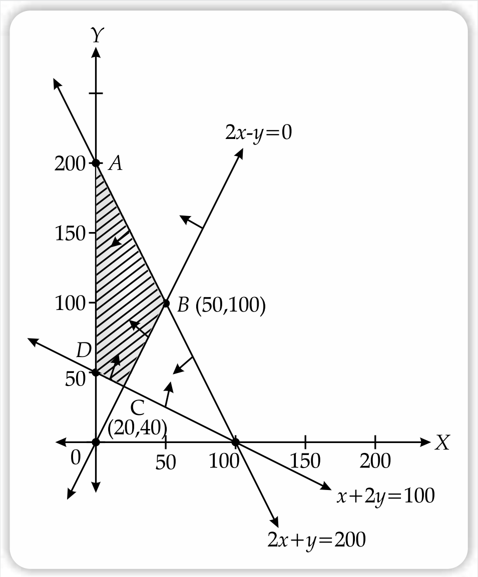
- Practice more problems on graphical solutions using a graph paper to avoid mistakes.

5. $Z = x + 2y$ and $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$, $x, y \geq 0$

For correct graph of three lines

For correct shading $ABCD$ **1**

$Z(A) = 0 + 400 = 400$



1

$Z(B) = 50 + 200 = 250$

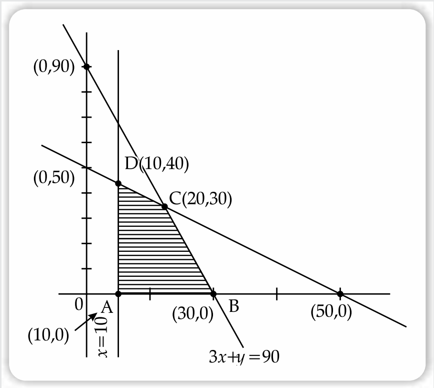
$Z(C) = 20 + 80 = 100$

$Z(D) = 0 + 100 = 100$

$\therefore$ Max ($Z = 400$) at $x = 0, y = 200$ **1**

[CBSE Marking Scheme 2017] (Modified)

7.



Correct graph of 3 lines **1**

Correct shade of 3 lines **1**

$Z|_{A\,(10,\,0)} = 40$

$Z|_{B\,(30,\,0)} = 120$

$Z|_{C\,(20,\,30)} = 110$

$Z|_{D\,(10,\,40)} = 80$

Maximum value of $Z = 120$ at (30, 0) **1**

[CBSE Marking Scheme, 2017] (Modified)

Long Answer Type Questions

2. (i) **1½**

Corner Points	$Z = 3x - 4y$
O(0, 0)	0
A(0, 8)	–32
B(4, 10)	–28
C(6, 8)	–14
D(6, 5)	–2
E(4, 0)	12

Max $Z = 12$ at E(4, 0)

Min $Z = -32$ at A(0, 8) **1**

(ii) Since maximum value of Z occurs at B(4, 10) and C(6, 8)

$\therefore \quad 4p + 10q = 6p + 8q$

$\Rightarrow \quad 2q = 2p$ **2**

$\Rightarrow \quad p = q$

Number of optimal solution are infinite. **½**

[CBSE SQP Marking Scheme 2020]

4. Let the company produce:

Goods $A = x$ units

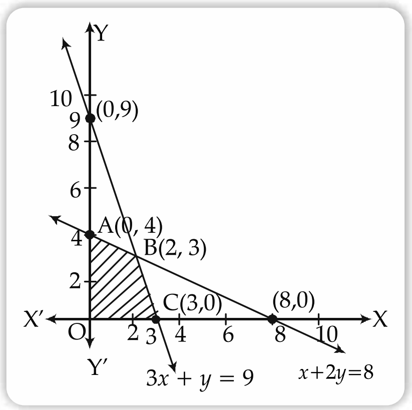
Goods $B = y$ units

then, the linear programming problem is:

Maximize profit: $Z = 40x + 50y$ (₹) **½**

Subject to constraints:

$$\left.\begin{array}{r} 3x + y \leq 9 \\ x + 2y \leq 8 \\ x, y \geq 0 \end{array}\right\}$$ **1½**



Correct graph: **2**

Corner point	Value of Z (₹)	
$A(0, 4)$	200	
$B(2, 3)$	230 (Max)	½
$C(3, 0)$	120	

∴ Maximum profit = ₹ 230 at:

Goods A produced = 2 units, Goods B produced = 3 units ½

[CBSE Marking Scheme, 2019] (Modified)

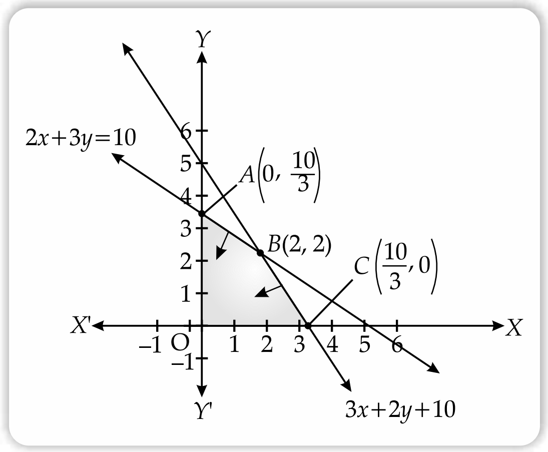
9. Let x and y be nut packages and bolt packages produced each day, respectively.

∴ LPP is Maximize $Z = 24x + 18y$ ½

Subject to $2x + 3y \le 10$

$3x + 2y \le 10$

$x, y \ge 0$ **1**



Vertices are

$A\left(0, \frac{10}{3}\right)$, $B(2, 2)$ and $C\left(\frac{10}{3}, 0\right)$ **2**

Points	$Z = 24x + 18y$
$A\left(0, \frac{10}{3}\right)$	$z = 0 + 60 =$ ₹ 60
$B(2, 2)$	$z = 48 + 36 =$ ₹ 84 (Max.)
$B\left(\frac{10}{3}, 0\right)$	$z = 80 + 0 =$ ₹ 80

Hence, 2 nuts & 2 bolts to be produced to get max. profit of ₹ 84. **1**

[CBSE Marking Scheme 2015] (Modified)

REFLECTIONS

- Do you think, farmers can use linear programming techniques to determine what crop they should grow and how much quantity they grow to get maximum profit?
- Can you apply linear programming problem to solve designing and manufacturing problems?
- Do you think transportation system rely upon linear programming for cost and time efficiency?

SELF ASSESSMENT PAPER - 05

Time: 1 hour **MM: 30**

UNIT-V

(A) OBJECTIVE TYPE QUESTIONS:

I. Multiple Choice Questions **[1×6 = 6]**

Q. 1. The maximum value of $Z = 5x + 7y$ subjected to constraints $x + y \leq 5, x \geq 0, y \geq 0$ is :

(A) 25 **(B)** 35 **(C)** 20 **(D)** 40

Q. 2. The point which does not lie in the half plane $4x + 6y - 28 < 0$ is

(A) (4, 2) **(B)** (2, 4) **(C)** (3, 1) **(D)** (1, 3)

Q. 3. Region represented by $x \leq 0, y \leq 0$ is

(A) first quadrant **(B)** second quadrant **(C)** three quadrant **(D)** fourth quadrant

Q. 4. If the value of the objective function Z can be increased or decreased indefinitely, such solution is called:

(A) Bounded solution **(B)** Unbounded solution

(C) Solution **(D)** None of these

Q. 5. The first step in formulating a LPP is :

(A) Identify any upper or lower bound on decision variables

(B) State the constraints as linear combinations of the decision variables

(C) Understand the problem

(D) Identify the decision variable

Q. 6. Constraints in an LP model represents:

(A) Limitations **(B)** Requirements

(C) balancing, limitations and requirements **(D)** all of above

II. Case-Based MCQs **[1×4 = 4]**

Attempt any 4 sub-parts from each questions. Each question carries 1 mark.

Read the following text and answer the questions on the basis of the same.

A train can carry a maximum of 300 passengers. A profit of ₹800 is made on each executive classes and ₹200 is made on each economy class. The IRCTC reserve at least 40 tickets for executive class. However, atleast 3 times as many passengers prefer to travel by economy class, than by executive class. It is given that the number of executive class ticket is ₹x and that of economy class ticket is ₹y.

Q. 7. The objective function of the LPP is :

(A) Max. Z = $800x + 200y$ **(B)** Max. Z = $200x + 800y$

(C) Min. Z = $800x + 200y$ **(D)** Min. Z = $200x + 800y$

Q. 8. Which among these is a constraint for this LPP?

(A) $x + y \geq 300$ **(B)** $y \geq 3x$ **(C)** $x \leq 40$ **(D)** $y \leq 3x$

Q. 9. Which among these is not a corner point for this LPP?

(A) (40, 120) **(B)** (40, 260) **(C)** (30, 90) **(D)** (75, 225)

Q. 10. Maximum value of z is

(A) ₹56,000 **(B)** ₹84,000 **(C)** ₹2,05,000 **(D)** ₹1,05,000

Q. 11. At which corner point the objective function has minimum value?

(A) (40, 120) **(B)** (40, 260) **(C)** (30, 90) **(D)** (75, 225)

(B) SUBJECTIVE TYPE QUESTIONS:

III. Very Short Answer Type Questions **[1×3 = 3]**

Q. 12. Represent the region by $-3x - 4y \geq -12$ graphically.

Q. 13. What is a solution of a LPP?

Q. 14. Define linear programming problem.

IV. Short Answer Type Questions–I **[2×3 = 6]**

Q. 15. A goldsmith manufactures necklaces and bracelets. The total number of necklaces and bracelets that he can handle per day is at most 24. It takes one hour to make a bracelet and half an hour to make a necklace. It is assumed that he can work for maximum of 16 hours per day. Further the profit on a bracelet is ₹500 and profit on a necklace is ₹200. Formulate this problem as a linear programming problem so as to maximize the profit.

Q. 16. The objective of a diet problem is to certain the quantities of certain foods that should be eaten to meet certain nutritional requirement at a minimum cost. The consideration is limited to milk, green vegetables and eggs, and to vitamins A, B, C. The number of milligram of each of these vitamins contained within a unit of each food and their daily minimum requirements along with the cost of each food is given in the table below :

Vitamin	Litres of milk	Kg. of vegetables	Dozen of Eggs	Minimum Daily Requirement
A	1	1	10	1 mg
B	100	10	10	50 mg
C	10	100	10	10 mg
Cost (in ₹)	20	10	8	

Formulate LPP for this diet problem.

Q. 17. A toy company manufactures two types of cars; a basic version car A and a deluxe version can B. Each doll of type B takes twice as long as to produce as one of type A, and the company would have time to make a maximum of 2,000 per day if it produces only the basic version. The supply of plastic is sufficient to produce 1500 cars per day (both A and B combined). The deluxe version requires a fancy accessories of which there are only 600 per day available. If the company makes profit of ₹5 and ₹10 per car respectively on car A and car B, then formulate this problem in LPP to maximize profit.

V. Short Answer Type Questions-II **[3×2 = 6]**

Q. 18. Solve the following LPP graphically:

Max. $Z = 5x + 3y$

Subject to $3x + 5y \leq 15$, $5x + 2y \leq 10$ and $x \geq 0, y \geq 0$

Q. 19. Solve the following LPP by graphical method:

Minimize $Z = 20x + 10y$

Subject to $x + 2y \leq 40$

$3x + y \geq 30$

$4x + 3y \geq 60$

and $x, y \geq 0$

VI. Long Answer Type Questions [1×5 = 5]

Q. 20. An oil company requires 12,000, 20,000 and 15,000 barrels of high-grade, medium grade and low grade oil, respectively. Refinery A produces 100, 300 and 200 barrels per day of high-grade, medium grade and low grade, respectively, while refinery B produces 200, 400 and 100 barrels per day of high-grade, medium grade and low grade oil, respectively. If refinery A cost ₹400 per day and refinery B costs ₹300 per day to operate, how many days should each be run to minimize costs while satisfying requirements.

❑❑